

Field theory models for variable cosmological constant

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Abstract

Anthropic solutions to the cosmological constant problem require seemingly unnatural scalar field potentials with a very small slope or domain walls (branes) with a very small coupling to a four-form field. Here we introduce a class of models in which the smallness of the corresponding parameters can be attributed to a spontaneously broken discrete symmetry. We also demonstrate the equivalence of scalar field and four-form models. Finally, we show how our models can be naturally embedded into a left-right extension of the standard model.

I. INTRODUCTION

Particle physics models suggest that natural values for the cosmological constant Λ are set by a high-energy scale, anywhere between 1 TeV and the Planck scale M_P (for reviews see [1–3]). The corresponding vacuum energy density ρ_Λ is then between $(1 \text{ TeV})^4$ and M_P^4 . On the other hand, recent observations indicate [4] that the actual value is $\rho_\Lambda \sim (10^{-3} \text{ eV})^4$, at least 60 orders of magnitude smaller. It is hard to explain why ρ_Λ should be so small. Even more puzzling is the fact that ρ_Λ appears to be comparable to the present matter density ρ_{m0} . The two densities scale very differently with the expansion of the universe, and it is very surprising that they nearly coincide at the present time.

To our knowledge, the only approach that can explain both of these puzzles is the one that attributes them to anthropic selection effects [5–14]. In this approach, what we perceive as the cosmological constant is in fact a variable that can take different values in different parts of the universe and it is assumed that the fundamental theory allows such a variation of the effective Λ . However, the particle physics models of variable Λ suggested so far appear to have some unnatural features.

In one class of models, the role of ρ_Λ is played by a slowly varying potential $V(\phi)$ of some scalar field ϕ which is very weakly coupled to ordinary matter. The simplest example is

$$V(\phi) = \rho_{bare} + \frac{1}{2}\mu^2\phi^2, \quad (1)$$

where ρ_{bare} is the “true” cosmological constant and it is assumed that ρ_{bare} and μ^2 have opposite signs. The two terms on the right-hand side of (1) nearly cancel one another in habitable parts of the universe. In order for the evolution of ϕ to be slow on the cosmological timescale, the mass parameter μ has to satisfy the condition [10]

$$|\mu| \ll 10^{-120} M_P^3 |\rho_{bare}|^{-1/2}. \quad (2)$$

The challenge here is to explain this exceedingly small mass scale in a natural way. The possibilities suggested so far include a pseudo-Goldstone field ϕ which acquires a potential through instanton effects [10,14], a large running of the field renormalization [12], and a non-minimal kinetic term with an exponential ϕ -dependence [13,14].

In the above mentioned class of models, the vacuum energy density ρ_Λ takes values in a continuous range. An alternative possibility is that the spectrum of ρ_Λ is discrete, as in the “washboard” potential model suggested in the early paper by Abbott [15]. A simple example of such a potential is

$$V(\phi) = -A \cos(2\pi\phi/\eta) + \epsilon\phi/\eta. \quad (3)$$

For $\epsilon \ll 2\pi A$, the potential has local minima at $\phi_n \approx n\eta$ with $n = 0, \pm 1, \pm 2, \dots$, separated from one another by barriers. The vacuum at $\phi = \phi_n$ has energy density

$$\rho_{\Lambda n} \approx n\epsilon. \quad (4)$$

Transitions between different vacua can occur through bubble nucleation.

Another version of the discrete model, first discussed by Brown and Teitelboim [16], assumes that the cosmological constant is due to a four-form field [17],

$$F^{\alpha\beta\gamma\delta} = \frac{F}{\sqrt{-g}} \epsilon^{\alpha\beta\gamma\delta}, \quad (5)$$

which can change its value through nucleation of branes. The total vacuum energy density is given by

$$\rho_{\Lambda} = \rho_{bare} + F^2/2 \quad (6)$$

and it is assumed that $\rho_{bare} < 0$. The change of the field across the brane is

$$\Delta F = q, \quad (7)$$

where the “charge” q is a constant fixed by the model. The four-form model has recently attracted much attention [18,13,19,21,14] because four-form fields coupled to branes naturally arise in the context of string theory.

In the range where the bare cosmological constant is almost neutralized, $|F| \approx |2\rho_{bare}|^{1/2}$, the spectrum of ρ_{Λ} is nearly equidistant, with a separation

$$\Delta\rho_{\Lambda} \approx |2\rho_{bare}|^{1/2}q. \quad (8)$$

In order for the anthropic explanation to work, $\Delta\rho_{\Lambda}$ should not exceed the present matter density,

$$\Delta\rho_{\Lambda} \lesssim \rho_{m0} \sim (10^{-3} \text{ eV})^4. \quad (9)$$

With $\rho_{bare} \gtrsim (1 \text{ TeV})^4$, it follows that

$$q \lesssim 10^{-90} M_P^2. \quad (10)$$

Once again, the challenge is to find a natural explanation for such very small values of q .

Feng, March-Russell, Sethi and Wilczek (FMSW) [19] have argued that the required small values of the charge q can naturally arise due to non-perturbative effects in M-theory. Their assumption is that some of the fundamental string theory branes can have an extraordinarily small tension $\sigma \lesssim 10^{-90} M_P^3$. The idea was to use D2-branes obtained by wrapping of k world-volume coordinates of a higher-dimensional Dp-branes ($p = 2 + k$) on a collapsing k -cycle. The volume of the cycle V_k then determines the effective tension of the resulting 2D-brane in lower dimension $\sigma \sim V_k$. It was assumed that non-perturbative quantum corrections may stabilize the volume at an exponentially small size, resulting in an exponentially small 2-brane tension. Then, assuming a typical tension-to-charge relation,

$$q \sim \frac{\sigma}{M_P}, \quad (11)$$

one arrives at the required value (10) of the brane charge.

The problem with this approach is that the small brane tension is not protected against quantum corrections below the supersymmetry breaking scale. Branes of the kind discussed by Feng *et. al.* originate as solitons of the fundamental theory, valid above the field theory cutoff M .¹ As a result, in the low energy effective field theory valid below the scale “M” these branes behave as fundamental objects, in the sense that the low energy observer cannot “resolve” their structure. Thus, the world-volume theory of these branes is a $2 + 1$ -dimensional field theory with a cut-off $\sim M$. In the absence of supersymmetry such branes would be expected to have a tension $\sigma \sim M^3$ and the assumption of $\sigma \ll M^3$ would be extremely unnatural. The existence of a spontaneously broken low-energy supersymmetry may ameliorate the situation a bit, but unfortunately not up to a sufficient level, as we shall argue below.

¹In our discussion everywhere M should be understood as the ultraviolet cut-off of the field theory, for which we take the string scale. We shall assume that the string coupling is of order one and none of the string compactification radii are large. Thus the theory below M is an effective four-dimensional theory and roughly $M_P \sim M$.

The problem is that, in the absence of an exact supersymmetry, the brane world-volume states will induce an unacceptably high brane tension through quantum loops. To demonstrate this, let us assume the most optimistic scenario, when the brane sector is only gravitationally coupled to the sector that spontaneously breaks supersymmetry. The lowest possible scale of supersymmetry breaking compatible with observations is somewhere around TeV energies. Thus, the mass splitting among the brane superfields induced by gravity will be at least

$$m_S^2 \sim TeV^4/M_P^2 \quad (12)$$

(this is the usual magnitude for the gravity-mediated supersymmetry breaking). Each brane superfield will induce a linearly-divergent contribution to the brane tension. For instance, at one-loop the contribution coming from a massless supermultiplet is

$$\Delta\sigma \sim m_S^2 \int \frac{d^3p}{p^2 - m_S^2} \sim m_S^2 \Lambda_{cut-off} \quad (13)$$

which we (at best) can cut off at the scale $M \sim M_P$. Thus, the resulting contribution to the brane tension from each pair of modes is expected to be

$$\Delta\sigma = TeV^4/M_P. \quad (14)$$

Thus, a single world-volume supermultiplet already gives an unacceptably large renormalization of the brane tension.

In addition, we have to sum over all the world-volume modes with masses below M . In the absence of an explicit model, it is hard to argue what the precise density of the modes is, and we will not speculate further on this issue. However, one may expect that an additional enhancement factor, can be as large as $\sim M/\sigma^{1/3}$ (since we expect the spacing of the modes in the world-volume theory to be set by the brane tension, the only scale in the low energy world-sheet Lagrangian). This would give the resulting tension to be something like $\sigma \sim TeV^3$. Although we cannot exclude a miraculous cancellation among the different modes, such a cancellation is not indicated by any symmetry and would be hard

to understand in an effective field theory picture. It is unlikely that string theory corrections can cure the problem since they set in only above the scale M , and such a conspiracy among high and low energy physics would constitute a violation of the decoupling principle.

From the above arguments, we expect that branes with very low tension and charge should be looked for among the effective field theory solitons. The tension of such solitonic branes is much better protected from quantum destabilization. As a simple straightforward example let us construct a solitonic brane with tension $\sigma \sim TeV^6/M^3$. Let ϕ be a chiral superfield with a superpotential

$$W = \frac{\phi^3}{3} \quad (15)$$

As above, let us assume that supersymmetry is spontaneously broken at the TeV scale in a sector that couples to ϕ only gravitationally.

To be more explicit, let S be a superfield which spontaneously breaks supersymmetry through the expectation value of its auxiliary (F_S)-component. By assumption $F_S \sim TeV^2$, which is the lowest possible phenomenologically acceptable scale. Then according to the standard picture, SUSY breaking in the ϕ -sector will be transmitted through gravity via couplings of the form

$$\int d^4\theta \frac{S^* S}{M_P^2} \phi^* \phi + \int d^2\theta \frac{S}{M_P} \phi^3 + .. \quad (16)$$

where θ is a superspace variable. The resulting soft mass of the ϕ scalar is $m_S^2 \sim \frac{|F_S|^2}{M_P^2} \sim TeV^4/M_P^2$ and the sign depends on the details of the theory. We shall assume that the sign is negative. Then the effective potential for ϕ can be written as

$$V(\phi) = -m_S^2 \phi^* \phi - (cm_S \phi^3 + \text{h.c.}) + |\phi|^4, \quad (17)$$

where c is a number of order one. Obviously, this system exhibits a spontaneous breaking of the discrete Z_3 symmetry ($\phi \rightarrow e^{i\frac{2n\pi}{3}}\phi$) and admits topologically stable solitonic brane solutions (domain walls) with tension $\sigma \sim m_s^3 \sim TeV^6/M^3$. These branes do not suffer from the quantum correction problems discussed above, due to the fact that all the integrals in

the brane world volume theory are cut off at the compositeness scale, which coincides with the brane tension scale m_s . Above this scale, one has to do computation in the full theory in which there is no renormalization of the mass parameters beyond the scale m_s , due to low-scale supersymmetry.

Although the above simple example demonstrates that the field theory branes with low tension are easily possible, it does not achieve our primary goal of generating very low charge branes, since the above branes are not coupled to any four-form field.² This will be our task in the following discussion.

Thus, in this paper we report on our search for field-theoretic models in which the smallness of q (or of μ in Eq. (1)) can be attributed to some symmetry. We found a class of models in which the value of q (or μ) can be made arbitrarily small. The charge-to-tension ratio q/σ can also be made as small as desired. As a byproduct of this research, we found a somewhat unexpected equivalence relation between scalar field and four-form models.

After outlining the general idea in Section II, we shall first discuss, in Section III, a simplified $(1 + 1)$ -dimensional version of our model. The equivalence between four-form and scalar field models is demonstrated in Section IV. Models in $(3 + 1)$ dimensions are discussed in Section V. In Section VI we illustrate how our models can be naturally embedded

²It was shown in [20] that domain walls of large- N supersymmetric $SU(N)$ -gluodynamics are automatically charged with respect to a (composite) three-form field. These are Z_N -walls formed by the gluino condensate, and both the tension and the charge of these objects are set by the $SU(N)$ -QCD scale Λ . However, there are two points that probably make these walls useless in the present context. First, the four-form field strength is the same on both sides of the wall, due to the fact that its change gets compensated by the change of the phase of gluino condensate. Second, these walls usually cannot exist if the supersymmetry breaking scale is $> \Lambda$. Therefore, according to our previous arguments, their charge can at best be $\sim TeV^4/M_P^2$, which is too high for our purposes.

into a left-right symmetric extension of the standard model. Our conclusions are briefly summarized and discussed in Section VII.

II. GENERAL PHILOSOPHY

Our idea is the following. 1) Branes with an extremely low four-form charge can appear in the form of solitons (domain walls) of an effective low energy theory. 2) The small value of the charge-to-tension ratio is natural in the sense that it can be arbitrarily suppressed by the symmetries of the model.

Before proceeding to specific examples, let us briefly discuss the main ingredients of our scheme: 1) a real scalar field, a , which can be thought of as the phase of a certain complex scalar field X with a nonzero vacuum expectation value $\langle X \rangle$; 2) a four form field $F_{\mu\nu\sigma\tau}$ which can be obtained from a three-form potential, $F_{\mu\nu\sigma\tau} = \partial_{[\mu} A_{\nu\sigma\tau]}$; 3) a scalar field ϕ , which spontaneously breaks a Z_{2N} discrete symmetry at the scale $\langle \phi \rangle$. The crucial assumption is that both scales $\langle \phi \rangle$ and $\langle X \rangle = \eta$ are well below the cutoff scale M . Having in mind a low energy SUSY, their natural value can be as low as TeV, which we shall adopt for definiteness³.

We require that the action be invariant under the following three symmetries: 1) Z_{2N} symmetry under which

$$\phi \rightarrow \phi e^{i\pi/N}, \quad a \rightarrow -a, \quad (18)$$

2) symmetry under the shift

$$a \rightarrow a + 2\pi\eta, \quad (19)$$

and 3) the three-form gauge transformation

³ In practice, even much higher scales can do the job, provided N is chosen to be large enough.

$$A_{\mu\nu\alpha} \rightarrow A_{\mu\nu\alpha} + \partial_{[\mu} B_{\nu\alpha]} \quad (20)$$

where $B_{\nu\alpha}$ is a two-form. In addition, we shall assume that there is an (at least) approximate global $U(1)$ symmetry $X \rightarrow e^{i\theta} X$, so that it is meaningful to talk of a as the phase degree of freedom. Thus, a can be regarded as a sort of a (pseudo)Goldstone particle. Note that even if $U(1)$ is explicitly broken by some non-perturbative Planck-scale-suppressed corrections, the mass of a will be suppressed by the powers of $\eta/M \sim TeV/M$, and is much smaller than the masses of other scalars in the theory, which we assume are around the TeV scale. Thus, below TeV energies we can integrate out the heavy quanta, such as ϕ and the radial part of X , and derive an effective low energy action for the remaining light fields, $A_{\mu\nu\alpha}$ and a . We shall assume that the low energy action includes all possible interactions that are compatible with the unbroken symmetries. Obviously, operators that are forbidden by spontaneously broken symmetries must appear suppressed (at least) by powers of the corresponding Higgs VEVs. Among such operators there is a mixing of a with the three-form potential,

$$\epsilon^{\mu\nu\alpha\beta} A_{\mu\nu\alpha} \partial_\beta a. \quad (21)$$

Since at high energies this operator is forbidden by the Z_{2N} -symmetry (as well as by an approximate global $U(1)$ -symmetry), in the low energy theory it should appear suppressed by the following factor

$$\frac{\langle \phi^N \rangle}{M^N} \epsilon^{\mu\nu\alpha\beta} A_{\mu\nu\alpha} \partial_\beta a + \text{h.c.} \quad (22)$$

Power N is dictated from the fact that ϕ^N is the lowest possible power of ϕ that makes up Z_{2N} -invariant in combination with a . Thus, the mixing can be arbitrarily suppressed by powers of TeV/M due to the symmetry reasons. This is enough to realize our program: the extremely small mixing coefficient automatically translates into an extremely small four-form charge of a -field domain walls. Like axionic domain walls, these walls must be present due to the 2π periodicity of the a -potential. Since a changes by 2π across the wall, each wall acts as a source for the four-form field. The resulting four-form charge is suppressed by the $a - A$ mixing (22) and is miniscule. We shall discuss this in more detail below.

Before proceeding, let us make a brief note. Below the energies comparable to the masses of either ϕ or the radial X quanta, $\langle\phi\rangle$ and $|\langle X\rangle|$ can be regarded as constants and the coupling (22) is a local gauge-invariant operator. For higher energies, however, one has to include additional momentum-dependent interactions. These become relevant for processes with external ϕ and X legs and must be included because of gauge invariance. Any interaction that will be responsible for inducing the above mixing term in the low energy theory will also generate momentum-dependent operators required by this gauge invariance (see the next section). These additional interactions, however, will play no role in our analysis, since they only contribute to processes with ϕ and X -quanta emission that are forbidden at energies of our interest.

III. A TOY MODEL IN $(1+1)$ DIMENSIONS

The field content of our $(1+1)$ -dimensional toy model is: the “electromagnetic” vector potential A_μ , a real scalar field a , an electrically charged (Dirac) fermion ψ , and a complex scalar field ϕ . The Lagrangian is

$$L = i\bar{\psi}\gamma_\mu D^\mu\psi + \bar{\psi}\gamma_\mu\psi\frac{\phi^N}{M^N}\epsilon^{\mu\nu}\partial_\nu a + \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu\phi)^2 - V(a, \phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \text{higher derivative terms}, \quad (23)$$

where D^μ is the usual covariant derivative with respect to the electromagnetic $U(1)$ group, $V(a, \phi)$ is some potential function, and $1/M$ is a constant of order one. Note that in $(1+1)$ dimensions M must be dimensionless, since the canonical dimensionalities of A_μ , ϕ and a are zero. For simplicity, here and below we use flat spacetime metric. Our sign conventions are $g_{00} = \epsilon^{01} = 1$.

We must stress that the interaction with fermions is introduced exclusively for the illustrative purpose. It allows us to trace explicitly how the gauge-invariant mixing operator arises in perturbation theory. In reality we will not necessarily rely on fermions, but assume that the mixing is induced by some perturbative or non-perturbative physics at the cut-off

scale.

As before, we require that the action be invariant under the following three symmetries: 1) $U(1)$ -gauge invariance, 2) Z_{2N} symmetry (18), and 3) the shift symmetry (19). Under these symmetries the function $V(a, \phi)$ is determined to depend on a and ϕ only through the invariants $\cos(a)$, ϕ^{2N} and $\phi^N \sin(a)$. The precise form of this function will be of no importance for us, as long as the vacuum expectation value $\langle \phi \rangle$ of ϕ is nonzero and $\langle \phi \rangle \ll M$. This expectation value spontaneously breaks Z_{2N} symmetry down to nothing, and some operators forbidden by this symmetry will be generated with strength suppressed by powers of the ratio $\epsilon = \frac{\langle \phi \rangle}{M}$.

Among such operators we shall be interested in A_μ - a mixing, which appears as a result of one-loop fermionic exchange. The corresponding operator has the form⁴

$$g \frac{\phi^N}{M^N} \epsilon^{\mu\nu} \partial_\mu a (g_{\nu\rho} - \frac{\partial_\nu \partial_\rho}{\partial^2}) A^\rho, \quad (24)$$

where g is a loop factor that includes a dimensionful gauge coupling. Shifting the ϕ field, we shall expand the theory around the vacuum state: $\phi \rightarrow \langle \phi \rangle + \phi(x_\mu)$. Performing integration by parts in the first term of the expansion, Eq. (24) can be written as

$$g \epsilon^N \epsilon^{\nu\mu} A_\mu \partial_\nu a + g \left(\frac{N \epsilon^{N-1} \phi(x_\mu)}{M^N} + \dots \right) \epsilon^{\mu\nu} \partial_\mu a (g_{\nu\rho} - \frac{\partial_\nu \partial_\rho}{\partial^2}) A^\rho, \quad (25)$$

where ellipses stand for the terms that contain higher powers of $\phi(x_\mu)$. These terms describe A_μ -to- a transition via emission of ϕ -particles, and therefore are not relevant at the energies below the mass of the ϕ -quanta. Thus, at low energies the only relevant operator is the first term in (25). An important fact is that this term is parametrically suppressed by powers of small quantity ϵ . After integrating out the heavy fields and the fermions, the relevant part of the low energy Lagrangian can be written as

$$L = \frac{1}{2} (\partial_\mu a)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(a) + g \epsilon^N \epsilon^{\nu\mu} A_\mu \partial_\nu a + \text{higher derivative terms} \quad (26)$$

⁴We thank A. Grassi for a clarifying discussion on this issue.

The potential $V(a)$ in (26) is the effective potential for a . As indicated above, the shift symmetry requires it to be an arbitrary periodic function of a . In the absence of $a - A$ mixing, the theory would have an infinite set of degenerate vacua at the minima of this potential, and standard topological arguments would imply that there must be domain wall configurations with a changing by $\Delta a = 2\pi$ across the wall.

Let us now see how this situation is affected by the $a - A$ mixing term in (26). The gauge field strength can be expressed as

$$F^{\mu\nu} = \epsilon^{\mu\nu} F. \quad (27)$$

The equation of motion for A field

$$\partial_\nu F^{\mu\nu} = -g\epsilon^N \epsilon^{\mu\nu} \partial_\nu a \quad (28)$$

then implies that the change of a across the wall is accompanied by a change of the field strength,

$$\Delta F = -g\epsilon^N \Delta a. \quad (29)$$

The vacua on the two sides of the wall will not generally be degenerate, due to this difference in the field strength. Higher-energy vacua will decay into lower-energy ones through nucleation of bubbles with the domain walls at their boundaries.

We thus see that the walls have acquired a charge

$$q = 2\pi g\epsilon^N, \quad (30)$$

which is suppressed by a power of the small parameter ϵ . A remarkable property of the above model is that, with a suitable choice of N , this charge can be made arbitrarily small.

IV. EQUIVALENT SCALAR FIELD MODEL

The scalar field equation, obtained by varying Eq. (26) with respect to a , is

$$\partial^2 a + V'(a) - g\epsilon^N F = 0. \quad (31)$$

Now, the field equation (28) for F can be integrated to yield

$$F = -g\epsilon^N(a - a_0), \quad (32)$$

where a_0 is an integration constant. Substituting this into (31), we see that the resulting equation for a is that for a scalar field with a potential

$$U(a) = V(a) + \frac{1}{2}\mu^2(a - a_0)^2, \quad (33)$$

where

$$\mu = g\epsilon^N. \quad (34)$$

Let us recall that $V(a)$ is periodic in a , $V(a + 2\pi) = V(a)$. Hence, the potential $U(a)$ is a “washboard” potential of the kind considered by Abbott [15].

The energy-momentum tensor for our model is

$$\begin{aligned} T_{\mu\nu} &= \partial_\mu a \partial_\nu a - \frac{1}{2}g_{\mu\nu} \partial_\sigma a \partial^\sigma a + g_{\mu\nu} V(a) + \frac{1}{2}g_{\mu\nu} F^2 \\ &= \partial_\mu a \partial_\nu a - \frac{1}{2}g_{\mu\nu} \partial_\sigma a \partial^\sigma a + g_{\mu\nu} U(a), \end{aligned} \quad (35)$$

where we have used Eq. (32). The last expression in (35) coincides with the energy-momentum tensor for a scalar field with a potential $U(a)$. We thus see that, as long as non-gravitational interactions of a and A_μ are negligible (apart from their interaction with one another), the model (26) is equivalent to that of a scalar field with a potential $U(a)$ given by Eq. (33). Tunneling between vacua with different values of F is replaced in this scalar model by tunneling between different minima of the washboard potential.

V. MODELS IN $(3 + 1)$ DIMENSIONS

We can easily generalize our model to $3 + 1$ dimensions. The main difference is that instead of the vector potential we shall consider a three-form field $A_{\mu\nu\alpha}$ and require the gauge invariance under $A_{\mu\nu\alpha} \rightarrow A_{\mu\nu\alpha} + \partial_{[\mu} B_{\nu\alpha]}$. The Lagrangian of interest then becomes

$$L = \frac{1}{2}\eta^2(\partial_\mu a)^2 - \frac{1}{4}F^2 - V(a) + g\eta^2 \frac{\phi^N}{M^N} \tilde{A}^\mu \partial_\mu a, \quad (36)$$

where

$$\tilde{A}^\mu = \epsilon^{\mu\nu\sigma\tau} A_{\nu\sigma\tau}, \quad (37)$$

a is dimensionless and η has dimension of energy (it has the meaning of the shift symmetry breaking scale). The last term in (36) should be understood as an effective low-energy interaction arising from some fundamental theory, with M being understood as the Planck scale. It is the lowest-dimension operator consistent with gauge, shift and Z_{2N} symmetries.

The shift symmetry (19) suggests that a might be an axion-like field arising as a phase of some complex scalar, $X = Re^{ia}$. Then $|\langle X \rangle| = \eta$ and the Z_{2N} symmetry is

$$\phi \rightarrow \phi e^{i\pi/N}, \quad X \rightarrow X^\dagger. \quad (38)$$

In terms of the fields X and ϕ , the last term in (36) can be written as

$$ig(\phi/M)^N \tilde{A}^\mu (X \partial_\mu X^\dagger - X^\dagger \partial_\mu X). \quad (39)$$

We shall assume that the Z_{2N} symmetry is spontaneously broken at a scale much below M . Having in mind a low energy supersymmetry, this breaking scale may be as small as TeV, without fine tuning. This would imply that the parameter $\epsilon = \langle \phi \rangle / M \sim 10^{-15}$.

As before, at low energies we can keep only a constant part in ϕ , in which case the last term in (36) reduces to

$$g\eta^2 \epsilon^N \tilde{A}^\mu \partial_\mu a. \quad (40)$$

The field equation for the four-form field (5) is

$$\partial_\mu F = g\eta^2 \epsilon^N \partial_\mu a, \quad (41)$$

and we obtain

$$\Delta F = g\eta^2 \epsilon^N \Delta a. \quad (42)$$

With a periodic potential $V(a)$, the model has domain wall solutions with a changing by $\approx 2\pi$ across the wall, and Eq. (42) indicates that these domain walls acquire a charge

$$q = 2\pi g\eta^2 \epsilon^N. \quad (43)$$

For $\epsilon \sim 10^{-15}$ and $\eta \lesssim M$, the condition (10) is satisfied with $N \geq 6$.

With ϵ so small, the effect of the field F on the structure of domain walls is negligible. The wall tension σ is determined solely by the potential $V(a)$ and is not suppressed by powers of ϵ . This is in contrast to M-theory based models, where q and σ are related by (11).

Following the same steps as in Section III, it can be shown that our model is equivalent to a scalar field model with a potential $U(a)$ given by (33),

$$U(a) = V(a) + \frac{1}{2}\mu^2(a - a_0)^2, \quad (44)$$

where now

$$\mu = g\epsilon^N \eta. \quad (45)$$

With μ suppressed by powers of ϵ , this washboard potential is of the type required in the Abbott's model [15].

An interesting version of the model is obtained by replacing the discrete shift symmetry by a symmetry with respect to arbitrary translations,

$$a \rightarrow a + \text{const.} \quad (46)$$

Then a is a Goldstone boson and $V(a) = 0$. The equivalent scalar model has the potential

$$U(a) = \frac{1}{2}\mu^2(a - a_0)^2. \quad (47)$$

This is of the same form as in the simple model (1) in which the effective vacuum energy density takes values in a continuous range. With $|\rho_{bare}| \sim (1 \text{ TeV})^4$, the condition (2) on μ is satisfied for $N \geq 6$.

VI. EMBEDDING INTO PARTICLE PHYSICS MODELS

In this section we would like to show that our model can be naturally embedded in well-motivated particle physics models. As an example we shall consider an embedding into a left-right symmetric extension of the standard model. We will see that the breaking of Z_{2N} symmetry can be associated with the spontaneous breaking of a left-right symmetry. We shall start by briefly reviewing a minimal left-right symmetric extension of the standard model [22]. The gauge group is $SU(3) \otimes SU(2)_L \otimes SU(2)_R \otimes U(1) \otimes P$ where P -parity interchanges left and right $SU(2)$ -subgroups. This will be identified with our parity symmetry. Left handed and right handed fermions form doublets of $SU(2)_L$ and $SU(2)_R$ groups respectively. Fermion masses are generated via coupling to a bi-doublet Higgs field (doublet under both $SU(2)$ -s) which we shall denote by H . From the point of view of the standard model subgroup, this bi-doublet field includes two electroweak Higgs doublets H_U and H_D , which give masses to “up” and “down” quarks (and leptons) respectively. For instance, the Yukawa couplings that give masses to quarks $Q_{L,R} = (U_{L,R}, D_{L,R})$

$$H\bar{Q}_L Q_R = H_U\bar{Q}_L U_R + H_D\bar{Q}_L D_R, \quad (48)$$

where we have written the $SU(2)_L \times U(1)_Y$ -decomposition. This coupling has an accidental Peccei-Quinn type global symmetry. The pseudo-Goldstone boson of this would-be symmetry is the phase difference of the electrically-neutral components of H_U and H_D doublets. This pseudoscalar state changes the sign under P -parity transformation. However, since this global symmetry is not exact, the would-be Goldstone boson is massive. Below our a field will be identified with this boson.

The left-right symmetry can be broken by introducing two doublet Higgs fields, H_L and H_R , that are doublets under left and right $SU(2)$ s respectively. One of them must develop an expectation value around the TeV scale (or above) thereby breaking the corresponding $SU(2)$ and left-right symmetry. The remaining $SU(2)$ subgroup then has to be identified with the electroweak $SU(2)_L$ group of the standard model.

A symmetry breaking in which only one of the doublets gets an expectation value can be achieved by an appropriate choice of the parameters in the Higgs potential. An important point, however, is that the presence of quartic terms in the potential is essential for the symmetry breaking. In a supersymmetric theory, such terms cannot be written at the renormalizable level, and as a result one has to introduce additional singlets, that are parity odd.⁵

The role of one such singlet in our case will be played by the ϕ field. The relevant couplings in the potential have the form

$$\frac{\phi^N}{M^{N-2}}(H_L^* H_L - H_R^* H_R) - m^2(H_L^* H_L + H_R^* H_R) + \lambda(H_L^* H_L + H_R^* H_R)^2 + \lambda'(H_L^* H_L H_R^* H_R) \quad (49)$$

where M, m are the mass scales around TeV and λ, λ' are positive constants. There are many other possible terms that are not essential for our discussion. The only requirement to the rest of the potential is that it forces ϕ to get a nonzero expectation value. Then, since the coupling with ϕ creates a left-right asymmetry, only one of the doublets is expected to get a VEV.

The coupling between ϕ and the doublets in (49) is non-renormalizable. However, it can be easily obtained by integrating out additional singlet fields of mass M and zero VEV. For instance, for $N = 6$ it is enough to introduce a single scalar field χ that under Z_{12} symmetry transforms as $\chi \rightarrow e^{i\pi/2}\chi$

$$\phi^3 \chi + \phi^{*3} \chi^* + M^2 \chi^* \chi + \chi^{*2} (H_L^* H_L - H_R^* H_R) \quad (50)$$

Integrating out χ we arrive at the effective non-renormalizable coupling given in Eq. (49).

⁵Note that without such singlet fields, one would have to include non-renormalizable quartic couplings suppressed by M_P in the superpotential. These, however, will create asymmetric minima at a very high scale, which is incompatible with the phenomenologically most interesting low energy LR-symmetric extension.

Below the TeV scale, the remaining effective low energy theory is the standard model coupled to our field a , via the pseudoscalar interaction. All other ingredients of our scheme (e.g. mixing with a three-form, etc.) are assumed to be unchanged. This completes the embedding of our model into a LR symmetric extension of the standard model.

VII. DISCUSSION

As explained in the Introduction, anthropic solutions to the cosmological constant problems require scalar field potentials with a very small slope or domain walls (branes) with a very small coupling to a four-form field. Here we introduced some models in which the smallness of the corresponding parameters can be attributed to a Z_{2N} symmetry, (18) or (38).

We note that, apart from the desired domain walls with small coupling to the four-form field, our models may have a variety of other topological defects. The breaking of the discrete Z_{2N} symmetry is accompanied by the formation of ϕ -walls such that the value of ϕ changes by a factor of $e^{i\pi/N}$ across the wall. $2N$ such walls can be joined along a ϕ -string, with the phase of ϕ changing by 2π around the string.

If the field a is the phase of a complex scalar, $X = Re^{ia}$, then the translation symmetry (46) is a $U(1)$ symmetry of phase transformations, $X \rightarrow e^{i\alpha}X$. When X gets an expectation value, this symmetry is broken and we expect global string solutions with a changing by 2π around a string. An interesting question is what happens to these strings at the Z_{2N} -breaking phase transition when ϕ gets an expectation value (we assume that it does so after X). With $\langle\phi\rangle = \text{const}$, the field strength F changes around the string by the amount (42) with $\Delta a = 2\pi$,

$$\Delta F = 2\pi g\eta^2 \epsilon^N, \tag{51}$$

suggesting that there should be a discontinuity along a sheet attached to the string. One way of resolving this obstruction is to assume that each X -string gets covered by a ϕ -string

with $2N$ walls attached to it, so that the cores of the two strings coincide.⁶ As we go around such a combined string, the sign of ϕ^N changes every time we cross a ϕ -wall, and with it the coupling between \tilde{A}^μ and a in (36) also changes sign. As a result, the change ΔF will have different sign in different sections between the ϕ -walls, and the overall change around the string will vanish. Another possibility is that the field F changes by the amount (51) inside a wall which gets attached to the X -string. Within such F -walls, the field ϕ should deviate from its vacuum value and the higher-derivative terms omitted in the Lagrangian (36) should become important. Which of the two options is realized may depend on the specific dynamics of the model.

Although the physics of topological defects in our model may be quite interesting, these defects are probably irrelevant for cosmology. The reason is that ϕ -walls would be disastrous if allowed to survive until present. The Z_{2N} symmetry breaking should therefore occur before the end of inflation, so that all X and ϕ strings and walls are inflated away and we are left only with a -walls at the boundaries of nucleating bubbles.

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